COMPARISON OF TWO SCHEMES OF COUNTER-COLLISION OF JETS HAVING DIFFERENT BERNOULLI CONSTANTS

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The problem of the collision of bimetallic plates during welding by explosion is examined in [1]. It is simulated by the diagram of the flow of a two-layered ideal incompressible weightless fluid jet. It is considered that the flow is planar stationary. If the densities of the metals forming the bimetallic plate are distinct, then the Bernoulli constants are dissimilar in the different layers and, consequently, the layer interfacial lines are lines of tangential velocity discontinuity. Such a combination of initial parameters governing the flow is possible for which the line of tangential velocity discontinuity is a bifurcating streamline. Then, as follows from the Bernoulli integral, the classical flow diagram with a critical point cannot be realized in which the velocity vanishes on both sides of the jet interfacial line. In this case, a flow diagram with a stagnation zone (Fig. 1a) proposed for problems with lines of tangential velocity discontinuity by Sedov [2] can be considered. The stagnation zone size and shape depend on the magnitude of the pressure p_0 given therein and its value can vary within definite limits.

If the Bernoulli constants of the interacting jets are identical, then as p_0 tends to the stagnation pressure p_* the size of the stagnation zone diminishes without limit and it shrinks to a point. The flow diagram with the stagnation zone here goes over continuously into a flow diagram with a critical point. The situation is analogous when the Bernoulii constants of different jets are distinct. On the basis of the impossibility of realizing a classical flow diagram with a critical point for different Bernoulli constants of different jets are distinct. On the basis of the impossibility of realizing a classical flow diagram with a critical point for different Bernoulli constants, the erroneous deduction is made in [1] that p_0 should differ from the p_* of a jet with a smaller Bernoulli constant by a certain quantity and, consequently, the stagnation zone size cannot be less than the specific one. Indeed, as is shown below, as $p_0 \to p_*$, the stagnation zone diminishes without limit even in the case of different Bernoulli constants and shrinks to a point while the flow diagram with the stagnation zone goes over into the flow diagram with a cusp (reentry) point of the boundary streamline ACD (Fig. 1b) [3]. The flow pattern displayed holds if the velocity head of the jet A is less than the velocity head of the jet B. The line of tangential velocity discontinuity L goes along the tangent to the rectilinear boundary AB at the point C, where the slope of the streamline BCD at the point C changes continuously while the slope of the streamline ACD at the point C undergoes a jump of 180°. The velocity of the jet A vanishes at the point C while the velocity of the jet B that has a greater head is different from zero at the point C.

1. Instead of the flow with the stagnation zone [1], let us study reversed flow obtained from the initial flow by replacing the velocity vector at each point by its opposite. We then arrive at a problem of the collision of jets flowing oppositely to each other along a rectilinear solid wall AB. In the general case the jet velocity heads can be distinct. Such a problem is solved in [4] by an iteration method similar to that elucidated in [3]. The solution is sought by conformal mappings of the complex potentials of the individual jets w^+ and w^- and the Zhukovskii functions ω^+ and ω^- on the parametric half-planes t and u with correspondence of the points indicated in Fig. 2a.

Let p_1, v_1, v_2 denote the pressure and velocity moduli on the free streamlines AD and BD, respectively; H_1, H_2 be the jet widths at infinity; ρ_1, ρ_2 be the fluid densities in the jets; v_0^+, v_0^- be the velocity moduli at the stagnation zone boundaries FC and CE; $\omega^+ = \ln(dw^+/v_1dz) = \ln(v^+/v_1) - i\theta^+$ and $\omega^- = \ln(dw^-/v_2dz) = \ln(v^-/v_2) - i\theta$ be the Zhukovskii

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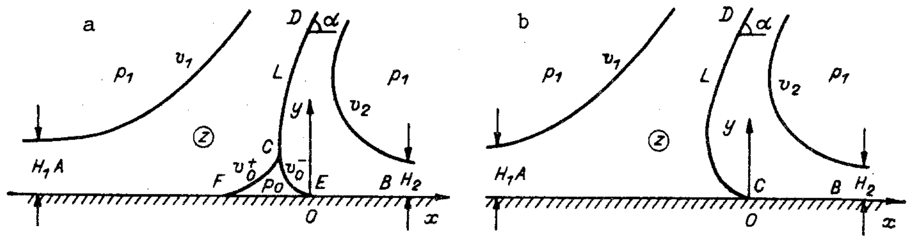

Fig. 1

functions; and t_0, u_0 be the coordinates of images of the points F and E on the parametric half-planes.

The desired solution is described by the system of equations

$$w^+(t) = \frac{q_1}{\pi}\ln\frac{1+t}{1-t}, \quad \frac{dw^+}{dt} = \frac{2q_1}{\pi(1-t^2)}; \tag{1.1}$$

$$w^-(u) = \frac{q_2}{\pi}\ln\frac{1-u}{1+u}, \quad \frac{dw^-}{du} = -\frac{2q_2}{\pi(1-u^2)}; \tag{1.2}$$

$$\omega^+(t) = \frac{1}{\pi i}\sqrt{\frac{t(t^2-1)}{t-t_0}}\left\{\ln\frac{v_0^+}{v_1}\int_{t_0}^{0}\sqrt{\frac{\tau-t_0}{\tau(\tau^2-1)}}\frac{d\tau}{\tau-t} + \int_0^1\sqrt{\frac{t_0-\tau}{\tau(\tau^2-1)}}\frac{\theta(\tau)\,d\tau}{\tau-t}\right\}; \tag{1.3}$$

$$\omega^-(u) = -\pi i + \frac{1}{\pi i}\sqrt{(u-u_0)(u-1)}\int_{-1}^{u_0}\frac{\ln\frac{v}{v_2}(\xi)}{\sqrt{(1-\xi)(u_0-\xi)}}\frac{d\xi}{(\xi-u)} \tag{1.4}$$

(q_1, q_2 are the bulk mass flow rates of the jets A and B, $q_1 = v_1H_1$, $q_2 = v_2H_2$, and θ is the slope of the velocity vector to the Ox axis).

Along the line of tangential velocity discontinuity L ($0 < t < 1$, $-1 < u < 0$)

$$\ln\frac{v^+}{v_1}(t) = -\frac{1}{\pi}\sqrt{\frac{t(1-t^2)}{t-t_0}}\left\{\ln\frac{v_0^+}{v_1}\int_{t_0}^{0}\sqrt{\frac{\tau-t_0}{\tau(\tau^2-1)}}\frac{d\tau}{\tau-t} + \operatorname{Re}\int_0^1\sqrt{\frac{\tau-t_0}{\tau(1-\tau^2)}}\frac{\theta(\tau)\,d\tau}{\tau-t}\right\}; \tag{1.5}$$

$$\theta^-(u) = \pi + \frac{1}{\pi}\sqrt{(1-u)(u_0-u)}\operatorname{Re}\int_{-1}^{u_0}\frac{\ln\frac{v^-}{v_2}(\xi)}{\sqrt{(1-\xi)(u_0-\xi)}}\frac{d\xi}{\xi-u}. \tag{1.6}$$

The constants t_0 and u_0 are determined from the equations

$$\ln\frac{v_0^+}{v_1}\int_{t_0}^{0}\frac{d\tau}{\sqrt{\tau(\tau^2-1)(\tau-t_0)}} + \int_0^1\frac{\theta(\tau)\,d\tau}{\sqrt{\tau(1-\tau^2)(\tau-t_0)}} = 0; \tag{1.7}$$

$$\frac{H_2}{H_1}\frac{v_0^+}{v_1}\int_0^{u_0}\frac{\sin\theta(\xi)\,d\xi}{1-\xi^2} - \frac{v_0^-}{v_2}\int_{t_0}^{0}\frac{\sin\theta(\tau)\,d\tau}{1-\tau^2} = 0. \tag{1.8}$$

The continuity boundary conditions for the pressure and the slope of the velocity vector should be satisfied on L

$$\ln\frac{v^-}{v_2}(u) = \frac{1}{2}\ln\left\{1 + \frac{\rho_1 v_1^2}{\rho_2 v_2^2}\left[\exp\left(2\ln\frac{v^+}{v_1}(t)\right) - 1\right]\right\}; \tag{1.9}$$

$$\theta^-(u) = \theta^+(t), \tag{1.10}$$

where $t \in [0, 1]$, $u \in [-1, 0]$, and t and u are connected by the relationship

$$\frac{du}{dt} = -\frac{H_1}{H_2}\frac{1-u^2}{1-t^2}\exp\left(\ln\frac{v^-}{v_2}(u) - \ln\frac{v^+}{v_1}(t)\right). \tag{1.11}$$

The solution of the problem under consideration is determined by three dimensionless parameters, the ratio between the jet widths being at infinity $h = H_2/H_1$, the ratio between the jet velocity heads being at infinity $\lambda = \rho_2 v_2^2/\rho_1 v_1^2$, and the number $\varkappa = (p_1 - p_0)/(1/2) \times \rho_1 v_1^2$ characterizing p_0. For definiteness, we consider that $\lambda \geq 1$.

The flow diagram with a stagnation zone for different velocity heads ($\lambda \neq 1$) allows for arbitrariness exactly as in the case of identical heads ($\lambda = 1$): p_0 can vary between p_1 on the free streamlines AD and BD and $p_* = p_1 + \frac{1}{2}\rho_1 v_1^2$ of the jet A with the smaller velocity head, which corresponds to a change in the dimensionless $\varkappa$ from 0 to −1, respectively. As $\varkappa \to 0$, the stagnation zone dimensions increase without limit.

Let us examine the behavior of the solution as $\varkappa \to -1$. The geometric flow patterns for h = 1, $\lambda = 1.1$, and different $\varkappa$ are displayed in Fig. 3. For comparison, flow patterns in the case h = 1, $\lambda = 1$ are presented for the same $\varkappa$ (the computations were performed by formulas in [5], because of symmetry half the flow is displayed, the linear dimensions are referred to the width H_1 of the jet A). The distribution of the pressure coefficient $c_p = (p - p_1)^{1/2}\rho_1 v_1^2$ along the solid rectilinear flow boundary is given by dashes. It is seen that in both cases $\lambda = 1$ and $\lambda = 1.1$) the dimensions of the stagnation zone diminish without limit as $\varkappa = -$ ($p_0 \to p_*$) and the zone shrinks to a point. If the flow diagram with the stagnation zone goes in the case of identical velocity heads here into a flow diagram with a critical point, in which the jet velocity on both sides of the jet interfacial line vanishes, then for different heads it will go over into a flow diagram with a reentry point (cusp) of the boundary streamline.

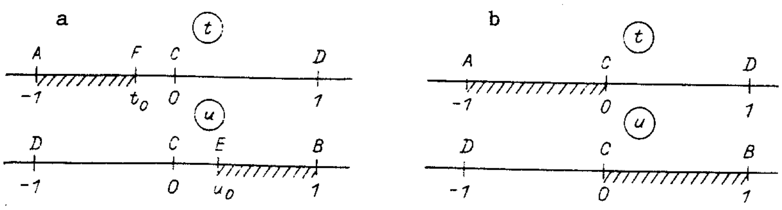

Fig. 2

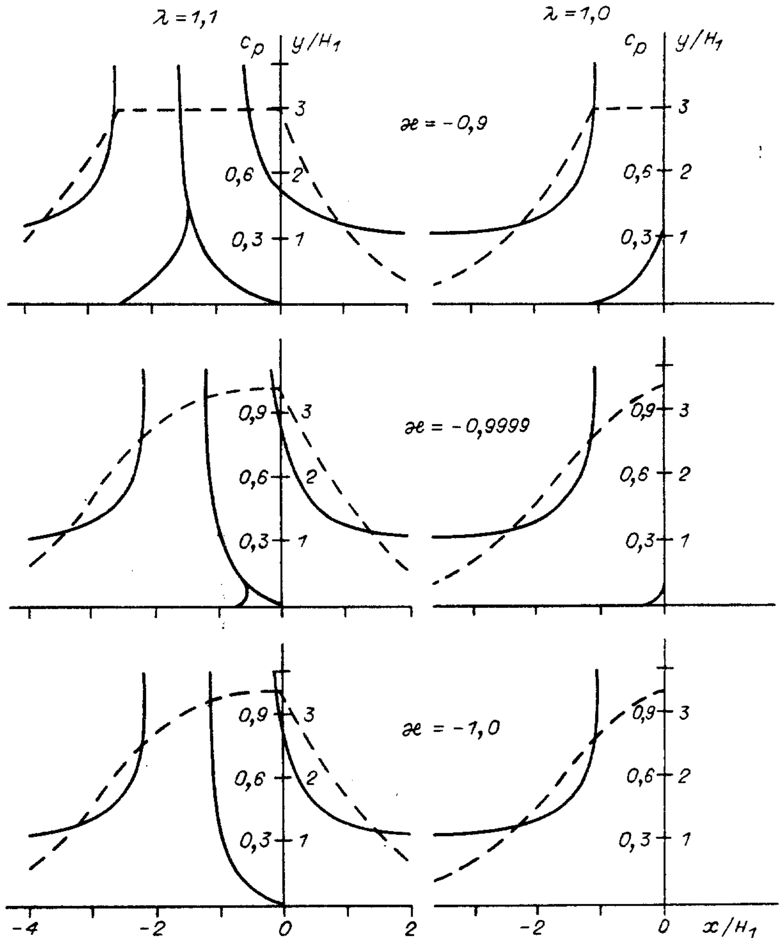

Fig. 3

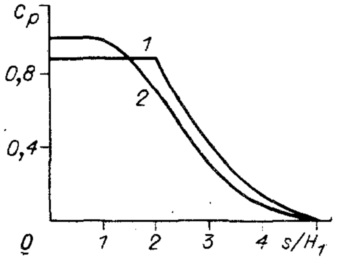

Fig. 4

As $\varkappa$ varies between the limits −1 and −0.99 (for $\lambda = 1.1$) the shape of the free streamlines and the streamline ECD (see Fig. 1a) remains practically unchanged although the stagnation zone dimensions vary substantially.

Graphs of the distribution of the pressure coefficient c_p along the streamline ECD are displayed in Fig. 4 as a function of the length s measured from the point E for different $\varkappa$ (line 1 is for $\varkappa = -0.9$). For $\varkappa = -1$-0.999 the graphs agree (line 2). Figs. 3 and 4 show that for flow diagrams with a reentry point ($\varkappa = -1$, $\lambda \neq 1$) the value of c_p is almost a constant equal to one near this point along both the streamline AC and the streamline CD, i.e., the value of the pressure is close to the value p_* of the jet A while the magnitude of the velocity modulus differs slightly from zero.

2. Now let us show that as $\varkappa \to -1$ the system of equations describing the flow in a diagram with a stagnation zone goes over into a system of equations describing the flow in a diagram with a cusp point.

The flow with a cusp (see Fig. 1b) is determined by two dimensionless parameters, $h = H_2/H_1$ and $\lambda = \rho_2 c_2{}^2/\rho_1 v_1{}^2$. We seek the solution for each jet separately by considering the pressure distribution (for the jet B) or the slope of the velocity vector (for the jet A) given on the separating streamline CD. Mapping the domain of variation of the complex potentials w^+ and w^- and the Zhukovskii functions ω^+ and ω^- on the parametric half-planes t and u with correspondence of the points indicated in Fig. 2b, we obtain a system of equations describing the flow under consideration. The equations for mapping $w^+(t)$ and $w^-(u)$ on t and u, as well as the boundary conditions on the line of tangential velocity discontinuity, are exactly the same as for a flow in a diagram with a stagnation zone: (1.1), (1.2), (1.9)-(1.11). The remaining equations have a form analogous to (1.3)-(1.6), where as is easily shown they are obtained from the latter by a passage to the limit under the conditions $t_0 \to 0^-$, $u_0 \to 0^+$, which are equivalent to the condition $\varkappa \to -1$.

As $\varkappa \to -1$ the velocity modulus $v_0{}^+$ on the stagnation zone boundary FC tends to zero; consequently, the arc length of FC also tends to zero, meaning $t_0 \to 0^-$. As follows from (1.8), here $u_0 \to 0^+$; consequently, the arc length of CE tends to zero.

Therefore, in the case of different Bernoulli constants the dimensions of the stagnation zone diminish without limit as $\varkappa \to -1$, it shrinks to a point and the flow diagram with the stagnation zone goes over into a flow diagram with a reentry point of the boundary streamline.

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